Angle Relationships in Triangles

The object of today's lesson is to find the measures of interior and exterior angles of triangles and to apply theorems about the interior and exterior angles of triangles.

Vocabulary

1. Auxiliary line — a line that is added to a figure to aid in a proof.

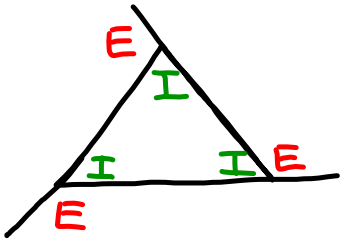
2. Corollary — "baby theorems"
a theorem whose proof follows directly from another theorem.
mother thm

3. Interior — the set of all points inside the figure.

4. Exterior — the set of all points outside the figure.

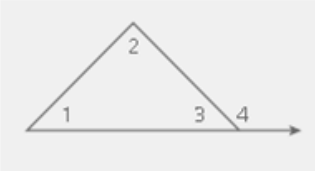
5. Interior angle — an angle formed by two sides of a triangle.

6. Exterior angle — the angle formed by one side of the triangle and the extension of an adjacent side.



7. Remote interior angle

an interior angle that is not adjacent to the exterior angle.



∠4 = exterior angle
∠1, ∠2, ∠3 = interior angles
∠3 and ∠4 = adjacent angles
∠3 = adjacent interior angle
∠1 and ∠2 = remote interior angles

Triangle Sum Theorem

The sum of the angles of a triangle is 180°

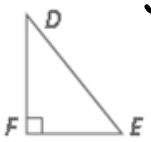
$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Corollaries to the Triangle Sum Theorem:

1. The acute angles of a right triangle are complementary.

HYPOTHESIS CONCLUSION

If $\triangle DEF$ is a right $\triangle$, then $\angle D + \angle E$ are complementary.



$m\angle D + m\angle E = 90°$

$m\angle D + m\angle E + m\angle F = 180$

$(m\angle D + m\angle E) + 90 = 180$

$-90 \quad -90$

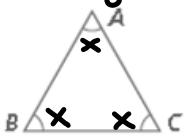
90

2. The measure of each angle of an equilateral triangle is 60°.

Equilateral $\triangle$ = 3 $\cong$ sides and 3 $\cong$ angles

HYPOTHESIS CONCLUSION

If $\triangle ABC$ is equilateral, then $m\angle A = m\angle B = m\angle C = 60°$



$m\angle A + m\angle B + m\angle C = 180$

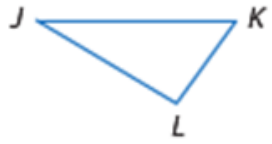
$x + x + x = 180$

$\frac{3x}{3} = \frac{180}{3}$

$x = 60$

3. There can be at most one right or obtuse angle in a triangle.

HYPOTHESIS CONCLUSION



If $\angle L$ is a right or an obtuse angle, then $\angle J$ and $\angle K$ must be acute angles.

$m\angle J + m\angle K + m\angle L = 180$

Say $m\angle L = 90$:

$(m\angle J + m\angle K) + 90 = 180$

$-90 \quad -90$

$m\angle J + m\angle K = 90$

The sum of these 2 angles must be 90.

Say $m\angle L = 108$:

$(m\angle J + m\angle K) + 108 = 180$

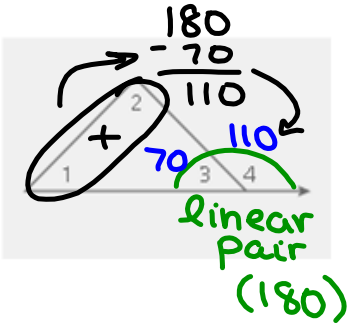
$-108 \quad -108$

$m\angle J + m\angle K = 72$

The sum of these 2 angles must be 72.

Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.



$m\angle 1 + m\angle 2 + m\angle 3 = 180$

$(m\angle 1 + m\angle 2) + 70 = 180$

$-70 \quad -70$

$m\angle 1 + m\angle 2 = 110 = m\angle 4$

$m\angle 4 = m\angle 1 + m\angle 2$

Third Angles Theorem

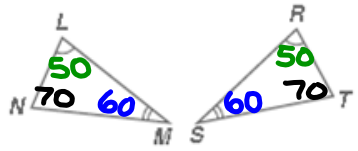
hypothesis

If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.

Conclusion

HYPOTHESIS

CONCLUSION



Prove:
$\angle N \cong \angle T$

Given: $\angle L \cong \angle R$ and $\angle M \cong \angle S$

$50 + 60 = 110$ $\quad 180 - 110 = 70 = m\angle N = m\angle T$

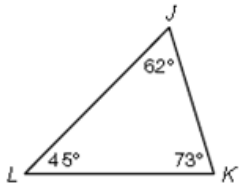
def. of $\cong$

EXAMPLE PROBLEMS

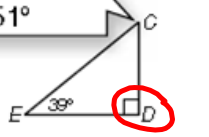
According to the **Triangle Sum Theorem**, the sum of the angle measures of a triangle is 180°.

$m\angle J + m\angle K + m\angle L = 62 + 73 + 45$

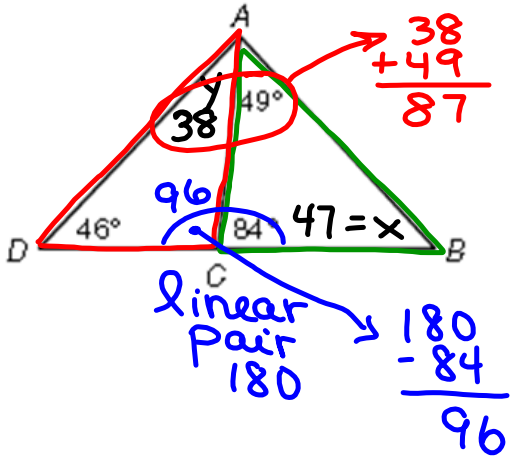
$= 180°$



The **corollary** below follows directly from the Triangle Sum Theorem.

Corollary	Example
The acute angles of a right triangle are complementary.	$m\angle C = 90 - 39$ $= 51°$  $m\angle C + m\angle E = 90°$

Use the figure for Exercises 1 and 2.



1. Find m∠*ABC*. $= 47$

$49 + 84 + x = 180$

$133 + x = 180$

$-133 \quad -133$

$x = 47$

2. Find m∠*CAD*. $= 38$

$46 + 96 + y = 180$

$142 + y = 180$

$-142 \quad -142$

$y = 38$

38 + 49 = 87

180 − 84 = 96

Use △*RST* for Exercises 3 and 4.

3. What is the value of x?

$m\angle R + m\angle S + m\angle T = 180$

$7x - 13 + 2x + 2 + 4x + 9 = 180$

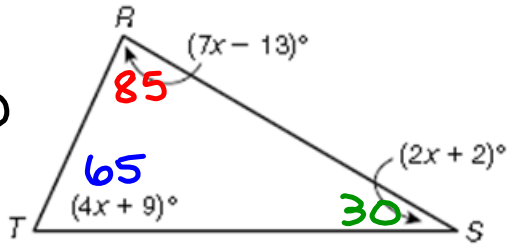
$13x - 2 = 180$

$+2 \quad +2$

$\frac{13x}{13} = \frac{182}{13}$

$x = 14$

4. What is the measure of each angle?



$m\angle R = 7(14) - 13$

$98 - 13$

85

$m\angle S = 2(14) + 2$

$28 + 2$

30

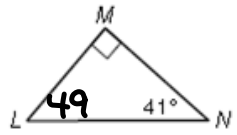
$m\angle T = 4(14) + 9$

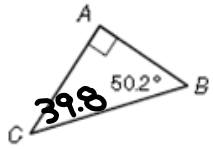
$56 + 9$

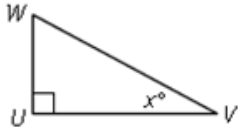
65

The acute angles of a right △ are complementary. (sum = 90)

What is the measure of each angle?







5. $\angle L$

$$\begin{array}{r} 90 \\ -41 \\ \hline 49 \end{array}$$

6. $\angle C$

$$\begin{array}{r} 90.0 \\ -50.2 \\ \hline 39.8 \end{array}$$

7. $\angle W$

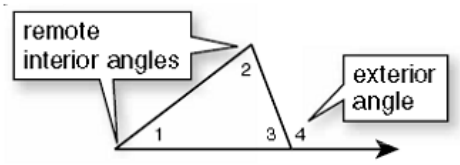
$m\angle W = 90 - \text{Given angle}$

$m\angle W = 90 - m\angle V$

$m\angle W = (90 - x)^\circ$

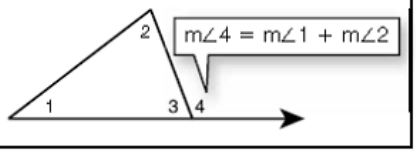
An **exterior angle** of a triangle is formed by one side of the triangle and the extension of an adjacent side.

∠1 and ∠2 are the remote interior angles of ∠4 because they are not adjacent to ∠4.



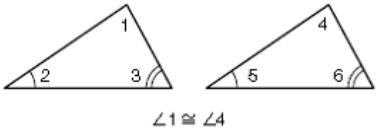
Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

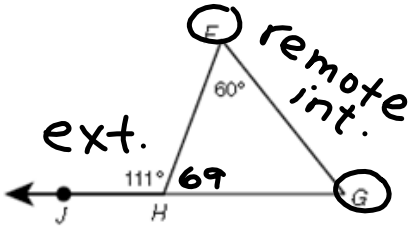


Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.



Find each angle measure. Exterior Angle Thm.

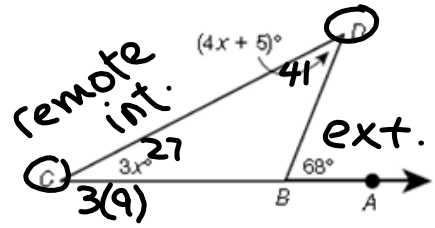


8. m∠G

$111 = 60 + m\angle G$

$-60 \quad -60$

$51 = m\angle G$



9. m∠D

$68 = 3x + 4x + 5$

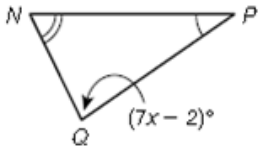
$68 = 7x + 5$

$-5 \quad -5$

$\frac{63}{7} = \frac{7x}{7}$ $x = 9$

$m\angle D = 4(9) + 5 = 36 + 5 = 41$

Find each angle measure. Third Angles Thm.

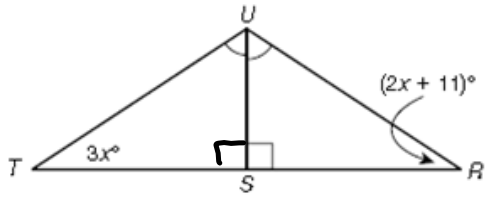


10. m∠M and m∠Q

$\angle L \cong \angle P$
$\angle K \cong \angle N$ $\longrightarrow$ $\angle M \cong \angle Q$
$m\angle M = m\angle Q$

$6x+10 = 7x-2$
$-6x+2 \quad -6x+2$
$12 = x$

$m\angle M = 6(12)+10 = 82$
$m\angle Q = 7(12)-2 = 82$



11. m∠T and m∠R

$\angle TUS \cong \angle RUS$
$\angle TSU \cong \angle RSU$ $\longrightarrow$ $\angle T \cong \angle R$
$m\angle T = m\angle R$

$3x = 2x+11$
$-2x \quad -2x$
$x = 11$

$m\angle T = 3(11) = 33$
$m\angle R = 2(11)+11 = 33$